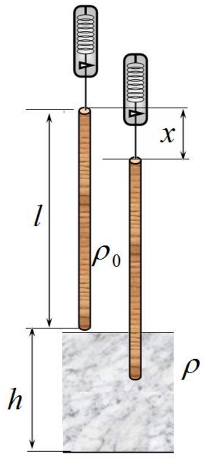
Problem 1. Warm-up Questions (10 points)

This problem consists of 3 independent questions

Problem 1.1. The sinking rod (4 points)



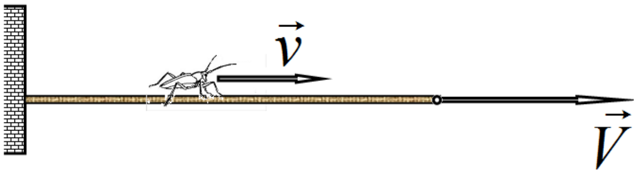
A uniform rod of length $l = 1{,}0\,m$ is made of material of density ρ_0. The radius of the cross section of the rod is much smaller than its length. The rod is suspended by one of its ends with a long, thin, weightless string from a dynamometer. When the rod is hanging freely in the air, the reading on the dynamometer is F_0. After that, the rod is slowly immersed in a wide tank of liquid of density ρ. The height of the liquid level in the tank is $h = \frac{l}{2}$. Friction of the rod against the horizontal bottom of the tank is negligible. Let us denote the height to which the rod sank as x ($x = 0$ when the lower end of the rod touches the surface of the liquid). The dynamometer readings when lowering the upper end of the rod to a distance x will be denoted by $F(x)$.

Plot on the forms in the answer sheets the graphs of the relationship between $f(x) = \frac{F(x)}{F_0}$ and the amount of the rod x being lowered. Solve the problem for two cases: a) the ratio of the densities of the liquid and the rod is $\eta = \frac{\rho}{\rho_0} = 2$; b) the density ratio is $\eta = \frac{\rho}{\rho_0} = \frac{1}{2}$.

At different stages of the immersion, dependencies $f(x)$ can be described by different formulas. Give these formulas for each of the stages, the range of changes of x in these stages. Draw the position of the rod at each step. Determine the number of stages in each case by your own.

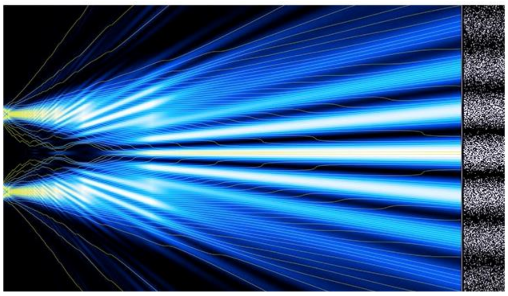
Problem 1.2. A bug on a cord (3 points)

A rubber cord of length $l_0 = 1{,}0m$ is attached to the wall at one end, the other end of the cord is pulled at a constant speed $V = 1{,}0\frac{m}{s}$. Simultaneously with the beginning of the cord stretching, a bug begins to crawl along the cord at a constant speed $v = 1{,}0\frac{cm}{s}$ (relative to the cord) starting from the wall.

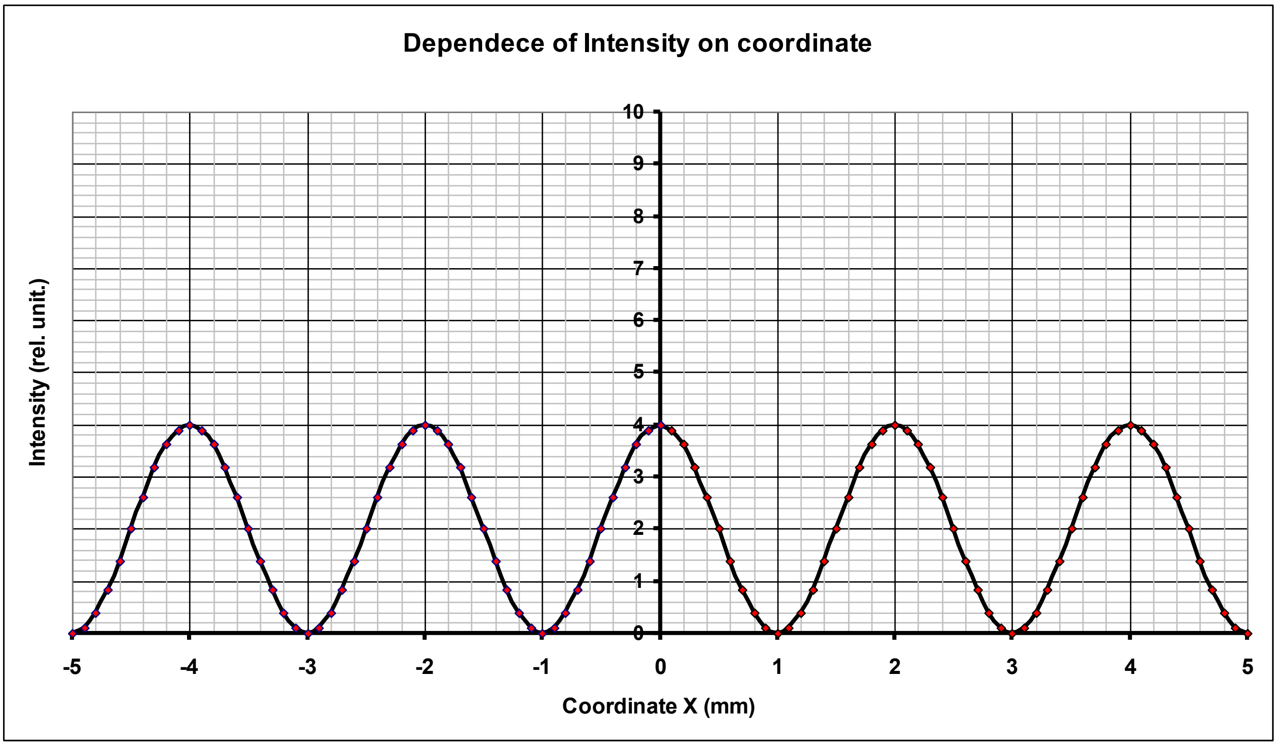


1.2.1 How many years does it take for the bug to reach the middle of the cord?
1.2.2 How many years does it take for the bug to reach the end of the cord?

Problem 1.3. Modernized Young's scheme (3 points)



In T. Young's interference scheme, a plane monochromatic wave is incident normally on a plane opaque plate. The plate has two identical narrow parallel slits at a short distance from each other. At a distance much greater than the distance between the slits, a flat screen is located parallel to the plate, on which an interference pattern is observed, consisting of stripes parallel to the slits in the plate. The dependence between the light intensity on the screen and the coordinate (the axis lies in the plane of the screen and is perpendicular to the observed stripes) is shown in the graph. This graph is also shown on the answer sheet.



1.3.1. Derive a formula which describes the distribution of light intensity on the screen $I_2(x)$. Using the given graph, determine the numerical values of the parameters of this formula.

A third slit is cut in the plate, parallel to the already existing slits and having the same width and located exactly in the middle between the original slits.

1.3.2. Derive a formula which describes the distribution of light intensity $I_3(x)$ on the screen in this case. Specify the numerical values for the parameters of this formula.

1.3.3. Plot on the form in the answer sheet a graph of the dependence between the light intensity on the screen and the coordinate x after the third slit has been cut in the plate.

Problem 2. Physics of popcorn (8 points)

The nutrient starch-containing tissue of a corn kernel has a hard shell(hull) on the outside and a soft content on the inside. It contains water. When the corn is heated, the water partially evaporates. At a certain temperature, the shell of the corn cannot withstand the pressure of the water vapor. It bursts, and the starch, softened by heat and pressure, expands, quickly turning into foamy structure, then cools and hardens, then cools and solidifies. Consequently, the corn kernels turn into popcorn flakes. To make an estimation, we can assume that the structure of the tissue is similar to a soap foam (spongy structure), in which the volume of bubble walls is negligibly small compared to the volume of pores.

It can be considered that the volume of the shell of the corn kernel remains unchanged up to the burst. For estimations, the corn kernels and the resulting popcorn flakes can be assumed to be spherical. The gas pressure inside the corn at which the shell bursts (at normal external atmospheric pressure $P_0 = 1{,}0\,atm = 1{,}0 \cdot 10^5\,Pa$), is called critical pressure P_{cr} .

After the burst of the shell, the process of expansion of gases contained in the soft tissues of a corn kernel can be considered as an adiabatic process. The equation for this process is:

$$PV^{\gamma} = const \qquad (1)$$

where γ - is the adiabatic exponent which in this case is equal to $\gamma = \dfrac{4}{3}$.

Part 1. Critical burst pressure (3.5 points)

In this part of the problem, you are asked to estimate in three different ways the critical pressure P_{cr} at which the shell (hull) of a corn kernel bursts. In addition to that, assume that the external pressure is equal to the normal atmospheric pressure.

1.1. The average radius of the corn kernels is $r_0 = 3{,}1\,mm$ and the average radius of the resulting popcorn flakes is $r_1 = 6{,}5\,mm$. Using this data, determine at what pressure P_{cr} inside the corn kernels the shell bursts.

1.2. The density of the corn kernels is $\rho_0 = 1{,}3 \cdot 10^3 \frac{kg}{m^3}$, and the density of the popcorn flakes formed is $\rho_1 = 0{,}16 \cdot 10^3 \frac{kg}{m^3}$. Using this data, determine at what pressure P_{cr} inside the corn kernels the shell bursts.

The corn kernel is covered with a thin and strong shell, the thickness of which is $h = 0{,}16\ mm$. The maximum mechanical stress that the shell material can withstand is approximately equal to $\sigma_c \approx 10 \cdot 10^6\ Pa$.

1.3. Determine what maximum internal pressure such a shell can withstand.

1.4. Specify what critical pressure value you will use in further calculations.

Part 2. Breaking temperature (3.5 points)

In this part of the problem you need to estimate the temperature t_{cr} to which the corn kernels should be heated (in degrees Celsius) so that they start to pop and form popcorn flakes. Consider that the initial temperature of the corn kernels is equal to room temperature $t = 20°C$.

2.1. Suppose that the pores of the soft tissues of corn kernels contain only dry air and no water vapor is formed when heated. Under this assumption, calculate the temperature t_{cr} to what the corn kernels should be heated so that they begin to burst.

2.2. In reality, soft tissues contain water, which evaporates when heated. Calculate the temperature t_{cr} to which corn kernels should be heated so that they begin to burst.

<u>Hint.</u> The dependence of saturated water vapor pressure P_S on temperature T is approximately described by the Clapeyron-Clausius equation

$$P_S = P_0 \exp\left[\frac{ML}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{2}$$

where $M = 18 \cdot 10^{-3} \frac{kg}{mol}$ is molar mass of water, $L = 2{,}3 \cdot 10^6 \frac{J}{kg}$ is latent heat of vaporization of water, $R = 8{,}3 \frac{J}{mol \cdot K}$ is universal gas constant.

Remainder: At normal atmospheric pressure $P_0 = 1{,}0\ atm = 1{,}0 \cdot 10^5\ Pa$ water boils at $t_0 = 100°C$ ($T_0 = 373K$) temperature.

2.3. Estimate how much of water must evaporate so that the pressure inside of the corn kernel reaches a critical value P_{cr}

Part 3. Gigantic popcorn (1 point)

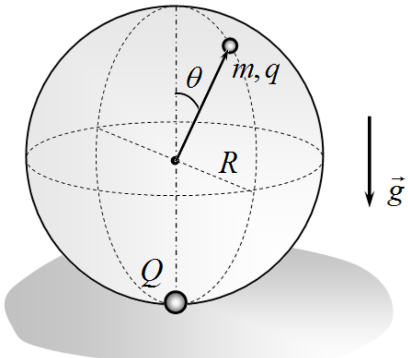
To increase the volume of popcorn flakes, the corn kernels are heated at a reduced external pressure P_1.

3.1. Estimate at what external pressure P_1 during the heating of the corn kernels, the average volume of popcorn flakes will be about 2 times greater than that of flakes formed at normal atmospheric pressure.

Note.
All numerical values of this problem were given from the following article:
Popcorn: critical temperature, jump and sound
Emmanuel Virot, Alexandre Ponomarenko
Published:06 March 2015, «Journal of the Royal Society Interface».
https://doi.org/10.1098/rsif.2014.1247

Problem 3. Inside the sphere

A point charge Q is fixed inside a non-conducting hollow sphere of radius R at its lowest point. A small ball (whose radius is much less than the radius of the sphere) of mass m and a charge q, which is also acted upon by gravity, can move along the inner surface of the sphere without friction. The signs of the charges are the same. The position of the ball on the inner surface of the sphere is determined by the angle θ measured from the vertical.



Polarization effects and friction can be neglected.

To reduce algebraic calculations and simplify formulas, we introduce a dimensionless parameter:

$$f = \frac{Qq}{16\pi\varepsilon_0 R^2}\frac{1}{mg} , \qquad (1)$$

which is the ratio of electrostatic force between charges (when the ball is at the top point) to the weight of the ball.

Part 1. General characteristics

1.1. Write down the expression for the total potential energy $U(\theta)$ of the ball as it moves from the top point as a function of the angle θ. Let us assume that the total potential energy of the ball at the top point is zero. Express your answer in terms of m, g, R, f.

1.2. Write down the expression for the absolute values of toques due to gravity $M_g(\theta)$ and electrostatic force $M_e(\theta)$ with respect to the center of the sphere as a function of the angle θ. Express your answer in terms of m, g, R, f.

1.3. In the answer sheet plot the schematic graphs of absolute values of torques $M_g(\theta)$ and $M_e(\theta)$ that act on the ball verses the deviation angle θ for $f = 1{,}0$, $f = 2{,}0$, $f = 3{,}0$.

1.4. In the answer sheet plot the schematic graphs of total potential energy $U(\theta)$ of the ball verses the deviation angle θ for $f = 1{,}0$, $f = 2{,}0$, $f = 3{,}0$.

<u>Note</u>. *While constructing schematic graphs, there is no need to carry out numerical calculations: indicate the ranges of increase and decrease, the presence of extrema.*

1.5. Calculate at what values of the parameter f the ball will not take off from the inner surface of the sphere during the motion.

1.6. Calculate at what values of the parameter f the ball can be in a stable equilibrium position at the top point of the sphere.

Part 2. The motion of the ball at $f = 1$

The moving ball was given such an electric charge that the dimensionless parameter became $f = 2$. Initially, the ball rests at the top point of the sphere and then begins to slide along its inner surface.

2.1. Determine the maximum deviation angle $\theta_{\max}$ of the ball during the motion.

2.2. Determine at what angle $\bar{\theta}$ the ball can be in a stable equilibrium position.

2.3. At what angles θ can the ball move along the horizontal circle inside the sphere?

Part 3. The motion of the ball at $f = 2$

The moving ball was given such an electric charge that the dimensionless parameter became $f = 2$. Initially, the ball rest at a certain point inside the sphere deviated by a small angle θ_0. The ball is released without initial velocity and it starts to oscillate in the vertical plane.

3.1 Find how the period of oscillation of the ball depends on the angle of initial deviation θ_0.

Note. You should get a formula that may contain an unknown dimensionless coefficient of proportionality.

Part 4. The motion of the ball at $f = 3$

The moving ball was given such an electric charge that the dimensionless parameter became $f = 3$. Initially, the ball rests at the top point of the sphere and then was given a small initial velocity v_0 directed along the sphere's surface.

4.1. Determine the period of small oscillations of the ball.

When the ball was at its lowest point, where the speed of the ball was zero, it was imparted a certain horizontal velocity v_1 along the inner surface of the sphere by an instant push.

4.2. At what value of v_1 the ball will move along the horizontal circle?